Workshop Noncommutative Geometry and Particle Physics

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Pointers for a discussion of the study group: **Higgs vacuum stability**

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The discovery of the Higgs with a mass of approximatively 125 GeV [1] can be considered a final confirmation of the standard model, but we may also hope that it may herald the opening of new physics. In this sense the fact that the mass of the Higgs is relatively "light" might point in this direction.

Possible discussion topics

In my opinion there are some topics which can be focus points of the discussion of the study group. I first indicate them schematically, and then I will give some details and references.

- The role of the running of the gauge couplings and the unification of their values at single scale for the spectral action.
- The meaning of the cutoff Λ, technical device to regularize or physical phase transition?
- Stability of the Higgs, the extra field σ and its role in the spectral action.
- Possibility to go beyond the one loop approximation.

Running of coupling constants

The various couplings of the standard model run with the energy, as dictated by the renormalization group. In the following the running analysis will be discussed in perturbation theory at one-loop. present technology enables calculation to two-loops in all cases [2–5], and three loops in some cases [6]. We will comment on the possible calculation with than one loop later on.

The runnings of the three coupling constants are given by the following equations, we skip the discussion on how to obtain this, the calculation is standard and can be found in any standard textbook on Quantum Field Theory:

$$\frac{dg_i(t)}{dt} = \beta_i(t), \quad \beta_i \equiv \frac{1}{16\pi^2} g_i^3 b_i, \quad t \equiv \log \frac{\mu}{GeV}. \tag{1}$$ RG

Where $i = 1, 2, 3$ are the U(1), SU(2) and SU(3) couplings respectively and μ is the renormalization scale. It is useful to define

$$\frac{1}{\alpha_i} \equiv \frac{4\pi}{g_i^2} \tag{2}$$

whose evolution is linear:

$$\frac{d}{dt}\left(\frac{1}{\alpha_i}\right) = -\frac{1}{2\pi} b_i \tag{3}$$

with

$$\begin{aligned} b_1 &= \frac{41}{6} \\ b_2 &= -\frac{19}{6} \\ b_3 &= -7 \end{aligned} \tag{4}$$

The values of the b_i's are given by the number and charges of the fermions. The two nonabelian interactions have a different sign form the U(1) coupling. At high energy they become asymptotically free. The abelian interaction has instead a Landau pole at very high energy, well above the Planck scale. At higher loops the β functions will depend in a nonlinear way from the other couplings, including the parameters of the Higgs and the Yukawa couplings. In order to establish the running low energy boundary conditions are necessary, they are experimental value and for Fig. 1 we have taken $g_1 = 0.358729$,

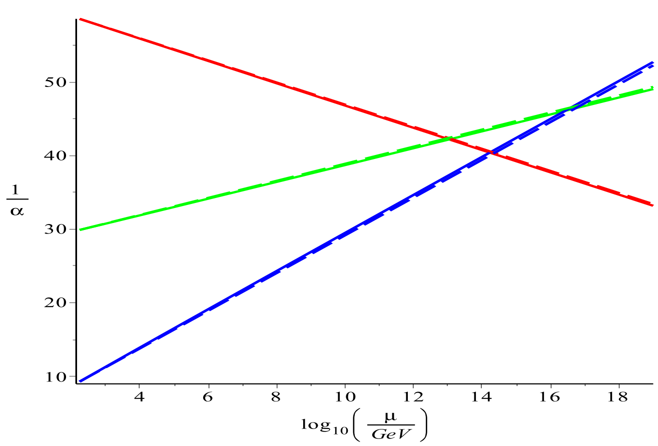

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Figure 1: *The running of α_i, the inverse of the gauge couplings. The dashed lines are the one loop approximation, solid lines two loop results. The α_i are in descending order as i increases.*

$g_2 = 0.648382$, $g_3 = 1.16471$ at the top mass scale. Of relevance for us is the fact that the three coupling constants almost coincide at a single scale. How "almost" lies in the eyes of the beholder. The three lines create a triangle, the values of $1/\alpha_i$ go from approximatively 40 to 50. These are pure numbers and therefore the total span is about 25% of the values. But the scale at which this happens goes from 10^{14} to 10^{17} GeV. A span of more than three orders of magnitude. Two and three loops calculations do not significantly alter these numbers.

This behaviour is the one given by the particles of the standard model. The presence of "new physics", be it in the form of new particles or new interactions will alter this. It is known for example that supersymmetry will change the slopes, due to the presence of supersymmetric partners, and that in some models the three couplings do coincide at a single scale. I do not know the present status of these models, also in view of the recent *LHC* results.

Unification scale and cutoff

The unification of the three couplings, or its lacking, is relevant for the spectral action:

$$S_B = \operatorname{Tr} \chi \left(\frac{D}{\Lambda} \right) \tag{5}$$

where $\chi(x)$ is a cutoff function. It is usually considered to be the characteristic function of the unit interval, or a smoothened version of it. Sometimes, to ease the heath kernel calculations [7, 8], might be considered to be a decreasing exponential. At any rate the cutoff of the eigenvalues of the Dirac operator D requires a ultraviolet scale. Performing the heat kernel expansion this means that the three couplings g_i's are equal at a sclae, which is natural to associate with Λ. As we have seen there is (in the absence of new physics) a single unification scale,and therefore one has to make a choice. What was done for the original prediction [9] was to set the scale at 10^{17} GeV, leading to the 170 GeV prediction. Later the whole range has been considered, leading to the prediction of a range of values. From the computational point of view the whole range does make a difference, although not a dramatic one, but from a conceptual point of view a single unification point is a requirement of the theory.

Some years ago the data seemed to indicate the presence of a single unification point around 10^{16} GeV. This was considered the point at which there would be a Grand Unified Theory (GUT), such as the ones based on a larger gauge group like SU(5), or SO(10). A GUT meant the presence of extra vector bosons, which mediate proton decay. At present the minimal SU(5) seems to be excluded by the limit on the proton lifetime. Other GUT's are still viable, but I think it is fair to say that they seem to be somewhat out of fashion.

There are variants of these unifications. For example (but my knowledge is limited) there have been studies on the possibility of a "strong" unification of all constants in the form of a pole (see for the example the review [10]). Another possibility is the fact that the unification happens at the Planck scale. This is natural if one considers that the unification signals a phase transition to a regime where quantum gravity cannot be ignored. The unification may be in the form of a “Universal Landau Pole” [11]. Another (opposite) possibility is a unification at the Planck scale in the form of asymptotic freedom (or better a zero of the couplings) [12]. In the context of the spectral action similar ideas leading to a common zero of the couplings at the Planck Scale are in [13].

One aspect worth discussion is *what happens at the unification point?*.

Even besides a single unification point, I find it difficult to believe that the three couplings start form vastly different values, along different routes, come together in a relatively small region, and then after having greeted each other keep on going, the weak and strong forces havong interchanged their strength, the U(1) coupling now the strongest of the lot, gravity becoming a player in the interaction game shortly afterwards. From a noncommutative geometry point of view it is quite natural to assume that the cutoff scale of the spectral action signifies some sort of phase transition. This is debatable, of course, and should be one of the points of the discussion.

Stability of the Higgs

Apart from the gauge couplings also the other constants of the standard model run. In particular let us discuss the evolution of the coefficient λ_H of the quartic term of the Higgs field. Its running is given by:

$$\begin{aligned}\frac{d\lambda_H}{dt} &= \frac{1}{16\pi^2}\left(24\,{\lambda_H}^2 - 6\,y^4 + \frac{3}{4}\,g_2^4 + \frac{3}{8}\,\left({g_2}^2 + {g_1}^2\right)^2\right. \\ &\quad \left. + \left(-9\,{g_2}^2 - 3\,{g_1}^2 + 12\,y^2\right)\lambda_H\right). \end{aligned} \tag{6}$$

and it depends on the gauge couplings, as well as y, the Yukawa couplings of the top quark[1]. This in turn runs with equation

$$\frac{dy}{dy} = \frac{1}{(4\pi)^2}\,y\,\left(-\frac{9}{4}\,{g_2}^2 - \frac{17}{12}\,{g_1}^2 - 8\,{g_3}^2 + \frac{9}{2}\,y^2\right). \tag{7}$$

The solution of these running with the boundary conditions given by experimental values are in Figs 2 and 3. We have used $y = 0.937982$, $\lambda_H = 0.125769$ for $M_H = 124GeV$ at the scale of the top mass $\mu = M_t = 172.9GeV$. These values are insensitive to M_H in the range $124 - 126$ GeV. The important aspect is the fact that λ_H becomes *negative* at a scale of the order of 10^10. Two loop calculations make the situation slightly worse. A negative λ_H means an instability and renders the whole model inconsistent.

One possible solution is that the unstable phase is actully a *metastable* phase [14,15], with an average lifetime vastly exceeding the life of the universe. This is based on the fact

[1] In principle it should depend on the Yukawa couplings of all fermions, but the one of the top, orders of magnitude larger that the others, dominates and renders the effect of the other negligible.

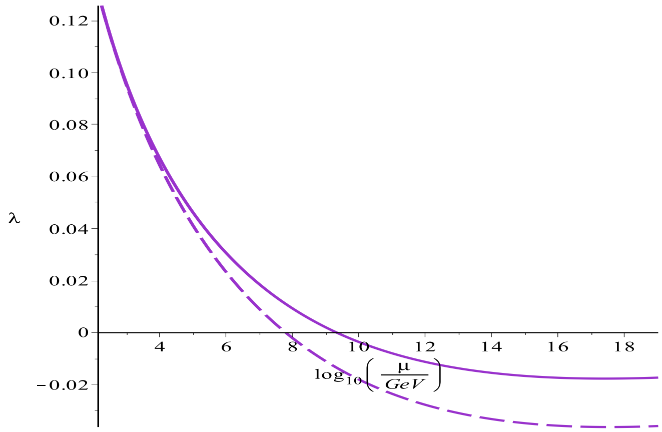

Figure 2: *The running of the quartic Higgs coupling. The dashed and solid lines one and two loop respectively as before.*

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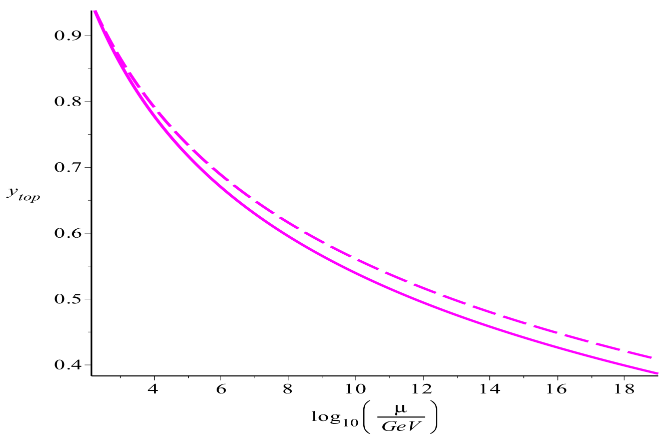

Figure 3: *The running of the top Yukawa coupling. he dashed and solid lines one and two loop respectively.*

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that the coupling becomes again positive at high scale. We will not discus this possibility since NCG has nothing to say so far in this respect, but we stress that the runnings present in all figure are with the *Big Desert* hypothesis, namely the absence of new physics from the scales of LHC to the Planck scale. In some sense the relatively lowmass of the Higgs hints at the fact that this big desert might be populated.

The σ field

The other possibility is therefore the presence of new physics which will alter the running. One such example is the role that can be played by an extra scalar field which couples to the Higgs, and to itself. This field has been proposed by various authors as stabilizer of the theory [16–20]. In the context of the spectral action this field has been proposed in [21] and related to the breaking of a extra U(1) symmetry, can bring down the mass of the Higgs to 126 GeV. It has also been also introduced in connection to the right handed neutrinos in [22], and is in fact essential in order to have a mass of the Higgs compatible with the experimental value. This field alters the running of λ_H due to the presence of the term $\lambda_{H\sigma}H^2\sigma^2$ in the Lagrangian. I do not write explicitly the full equations of the renormalization group, which can be found in [22].

The origin of this field has to do with the presence of right handed neutrino. The discovery of neutrino masses implied that right handed neutrinos, originally absent from the standard model, had to exist. From the NCG and spectral action point of view this meant an enlargement of the Hilbert space, and of the internal Dirac operator. In [9] is shown how the requirement that the internal, finite dimensional, algebra satisfies the formal requirement of being a "noncommutative manifold", poses stringent requirement of the internal algebra. The algebra must be of the kind

$$\mathcal{A}_{\mathcal{F}} = \mathbb{M}_a(\mathbb{H}) \oplus \mathbb{M}_{2a}(\mathbb{C}) \tag{8}$$ ncmanifold

with a an integer, $\mathbb{H}$ the real quaternionic algebra, and $\mathbb{M}$ the albebra of matrices over the complex or the quaternions respectively. The standard model algebra $\mathbb{C}\oplus\mathbb{H}\oplus\mathbb{M}_3(\mathbb{C})$ satisfies this requirement, as it emerges as a reduction of the algebra $\mathcal{A}_{\mathcal{F}}$ with $a = 2$ after the imposition of chirality and the first order condition:

$$[[D, a], JbJ] = 0 \ \ \forall A, b \in \mathcal{A} \tag{9}$$

The Dirac operator in the presence of right handed neutrinos takes into account the presence of a large Majorana mass, leading to the see-saw mechanism.

The bosonic fields emerge from the spectral action emerge as one forms, i.e. as elements of the form $\sum_i a_i[D, b_i]$. This is true for gluons, W and B field, and the Higgs field. It is an important conceptual success of this approach. The Higgs is completely on a par with the intermediate vector bosons, and is a scalar just because it relates to the fluctuations in the internal noncommutative space, and therefore has different properties under transformations of spacetime.

There is more than one position on the Dirac operator where a term with a Majorana mass may go. The problem is that however if one requires the first order condition then the only nonzero elements of the internal part of the Dirac operator D_F. The problem is that not all these will still satisfy the first order condition, and those which do satisfy it, give vanishing one form. Hence the σ field does not appear as a one form in this model if one requires *the first order condition* and at the same time considers *bosonic fields as coming from one-forms.*

One is therefore left with the task of explaining how this field may arise without violating the first order condition, or any other condition. In [**?**] the problem was circumvented considering the element of D_F to be a field from the start, thus putting it on a different footing from the other fields.

Two solutions for this problem have been proposed. They are not necessarily incompatible, and both point to an enlargement of the symmetries. In [23, 24] the proposal is to do without the first order condition, or rather to allow *the fluctuations to violate the first order condition*. This leads to a Pati-Salam symmetry, and the presence of the σ field which appears as the field which breakes the left-right symmetry to the electroweak one. There are other consequences of this, the Higgs appear as a composite field, alebit as an unconventional one. In [25] it was considered the possibility of a *Grand Symmetry* obtained from (8) in the case $a = 4$. In this case it is possible to have the larger algebra act on the same Hilbert space, which however cannot be seen as the product of four dimensional spinors times an internal 64 dimensional space, 64 being the number of fermionic physical degrees of freedom in the standard model. The grand algebra acts on the full Hilbert space (which takes into account fermion doubling [26, 27]). Also the Dirac operator has now more possibilities. Among those there are elements in which a Majorana neutrino mass can find place, still satisfying the first order condition, and give rise as one form to the field σ with the right properties. THe Grand algebra in this case has also some features of some sort of Pati-Salam model, and the field σ breaks the left-right symmetry. In this respect the two models might work together in synergy.

The σ field, its origins and its uses is certainly another discussion topic. A better understanding of it might lead to an enhanced predictive power if one can fix its parameters from experiment or other considerations. @@@

How many loops?

It is usually said that heath kernel calculations are one-loop calculations. This is not precisely correct. There are no Feynman diagrams in the expansion, what is usually meant is that the calculation is at first order, and since it is a semiclassical expansion, one could consider it to be at first order in $\hbar$, just as it is possible to see that a loop expansion corresponds to a similar expansion. In reality what happens is that the heath kernel gives the one-loop effective action,and this can be used to make useful prediction at that level. The two expansions are however not the same thing. For the standard model there is substantial quantitative difference between one and two loops [2–5], while there does not seem to be much of a difference with the addition of a third loop [6]. The situation in going to higher loops becomes however quickly rather complicated, for example in some cases results are renormalization scheme dependent. As far the heat kernel is concerned, it seems to be very difficult to go beyond one loop [8].

One can have two different points of view with respect to the spectral action and its heath kernel expansion. On one side one can consider the spectral action to be the "real action", and the heat kernel an approximation. This is the point of view taken in [28], or one can consider the heat kernel expansion of the spectral action to be the action with which one makes predictions, as in [29]. The two approaches are not equivalent and their differences are worth a discussion.

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