GAUGE THEORY AND NONCOMMUTATIVE GEOMETRY

RADBOUD UNIVERSITY NIJMEGEN

4–8 APRIL 2016

All talks will take place in lecture room HG00.303 of the Huygens building.

Monday 4th April

09.00-09.30	Registration and coffee/tea
09:30-10:30	José Gracia-Bondía: *Light-like string localized quantum fields*
10:30-11.00	Coffee/tea break
11:00-12:00	Latham Boyle: *The standard model as a differential graded super-algebra*
12:00-13:30	Lunch break (3rd floor, wing 7/8)
13:30-14:30	Fedele Lizzi: *Wick rotation, fermion doubling and Lorentz symmetry for the spectral action*
14:30-15:00	Coffee/tea break
15:00-16:00	Mairi Sakellariadou: *Spectral action and cosmology*
16:00-17:00	Ali Chamseddine: *Unification of gravity and gauge interactions*
17:00-18:00	Drinks, refreshments and piano music (by Olga Pashchenko)
18:00-19:00	**Public lecture by Alain Connes** (HG00.307) *'Geometry and the Quantum'*
19:00-19:30	Further refreshments and piano music (by Olga Pashchenko)

Tuesday 5th April

09:30-10:30	Shane Farnsworth: *The underlying algebraic structure behind the standard model of particle physics*
10:30-11.00	Coffee/tea break
11:00-12:00	Patrizia Vitale: *The Gribov problem in noncommutative QED*
12:00-13:30	Lunch break (3rd floor, wing 7/8)
13:30-14:30	Thierry Masson: *Generalized connections and Higgs fields on Lie algebroids*
14:30-15:00	Coffee/tea break
15:00-16:00	Giovanni Landi: *Line bundles over noncommutative spaces*
16:00-17:00	Francesca Arici: *Cuntz-Pimsner algebras and mapping cone exact sequences*
17:30-21:00	Conference dinner

Wednesday 6th April

09:30-10:30	Jens Kaad: *Morita equivalences of spectral triples*
10:30-11.00	Coffee/tea break
11:00-12:00	Bram Mesland: *Hecke operators and K-homology of arithmetic groups*
12:00-13:30	Lunch break (3rd floor, wing 7/8)
13:30-14:30	Magnus Goffeng: *Detecting regularity using cyclic cocycles and singular traces*
14:30-15:00	Coffee/tea break
15:00-16:00	Peter Hochs: *A fixed point theorem on non-compact manifolds*
16:00-17:00	Victor Gayral: *From equivariant quantization to locally compact quantum groups*

Thursday 7th April

09:30-10:30	Nadir Bizi: *The NC standard model: Lorentzian signature and NC differential forms*
10:30-11.00	Coffee/tea break
11:00-12:00	Michał Eckstein: *Hidden causal structure of gauge theories*
12:00-13:30	Lunch break (3rd floor, wing 7/8)
13:30-14:30	Koen van den Dungen: *The fermionic action*
14:30-15:00	Coffee/tea break
15:00-16:00	Farzad Fathizadeh: *Modular forms in the spectral action of Bianchi-IX gravitational instantons*
16:00-17:00	John Barrett: *The noncommutative geometry of defects*

Friday 8th April

09:30-10:30	Francesco D'Andrea: *The standard model in noncommutative geometry: fermions as internal Dirac spinors?*
10:30-11.00	Coffee/tea break
11:00-12:00	Anna Pachoł: *Hopf algebroids, Drinfeld twists and quantum space-times*
12:00-13:30	Lunch break (3rd floor, wing 7/8)
13:30-14:30	Tomasz Brzeziński: *Noncommutative geometry of generalized Weyl algebras*

List of Abstracts

Francesca Arici: Cuntz-Pimsner algebras and mapping cone exact sequences.

Cuntz-Pimsner algebras are universal C^*-algebras associated to injective C^*-correspondences and they provide a unifying framework for a class of examples including crossed products by the integers, Cuntz-Krieger algebras, graph algebras and quantum principal circle bundles. In this talk I will provide, for the case of bi-Hilbertian bimodules of finite Jones-Watatani index, an explicit isomorphism between the Cuntz-Pimsner exact sequence and the exact sequence for the mapping cone of the inclusion of the scalars into the Cuntz-Pimsner algebra, which is realised at the level of unbounded KK-cycles.

John Barrett: The non-commutative geometry of defects.

A diagrammatic calculus for finite real spectral triples is introduced. This calculus relates the axioms of non-commutative geometry with geometric properties of the diagrams. The diagram calculus determines a two-dimensional topological quantum field theory with defects. Conversely, defects in a TQFT determine a generalised notion of non-commutative geometry.

Nadir Bizi: The NC standard model: Lorentzian signature and noncommutative differential forms.

We first show how non-commutative differential forms can be used to refine the non-commutative standard model. We then redefine spectral triples using Krein spaces instead of the usual Hilbert spaces. By drawing a parallel with Clifford algebras, we show that the representation space of spectral triples should be an indefinite inner product space later turned into a positive definite one, and not the other way around. We study the properties of indefinite inner product spaces relevant to spectral triples.

Latham Boyle: The standard model as a differential graded super-algebra.

I will describe how the basic data $\{A, H, D, \gamma, J\}$ in the spectral triple formulation of NCG may be naturally repackaged into an "Eilenberg algebra" B (a particularly simple type of super-algebra). From the simple requirement that B is a differential graded star-algebra (or $*$-DGA), one then recovers nearly all of the traditional axioms governing the spectral triple, as well as a few novel constraints. When we apply our formalism to the spectral triple traditionally used to describe the standard model of particle physics, we find that these new constraints each correspond to a physically meaningful and phenomenologically correct constraint on the geometry. I will explain how this formalism is related to, but greatly improves upon, a proposal we made in 2014.

Tomasz Brzeziński: Noncommutative geometry of generalised Weyl algebras.

Generalised Weyl algebras over a polynomial ring in one variable can be understood as coordinate algebras of noncommutative or quantised surfaces. Perhaps the best known examples of such algebras are quantum standard Podleś spheres. Further examples include quantum teardrops and other quantum weighted projective lines. In this talk we describe principal circle bundles (or strongly integer-graded algebras) and the associated line bundles over generalised Weyl algebras, study differential structures both on the total and the base spaces and present algebraic aspects of the construction of Dirac operators with real structure of KO-dimension two.

Ali Chamseddine: Unification of gravity and gauge interactions.

The tangent group of the four dimensional space-time does not need to have the same number of dimensions as the base manifold. Considering a higher-dimensional Lorentz group as the symmetry of the tangent space, we unify gravity and gauge interactions in a natural way. The spin connection of the gauged Lorentz group is then responsible for both gravity and gauge fields, and the action for the gauged fields becomes part of the spin curvature squared. The realistic group which unifies all known particles and interactions is the $SO(1,13)$ Lorentz group whose gauge part leads to $SO(10)$ grand unified theory. I briefly discuss the Brout-Englert-Higgs mechanism which breaks the $SO(1,13)$ symmetry first to $SO(1,3)\times SU(3)\times SU(2)\times U(1)$ and further to $SO(1,3)\times SU(3)\times U(1)$. The spectral action associated with the Dirac operator is also discussed.

Francesco D'Andrea: The standard model in non-commutative geometry: fermions as internal Dirac spinors?

On a spin manifold M, an algebraic characterization of the module of Dirac spinors (sections of the spinor bundle) is as the Morita equivalence bimodule between the algebra of functions on M and the Clifford algebra bundle. This condition admits a natural generalization to (non-commutative) spectral triples and imposes some constraints on the form of the Dirac operator. I will report on a recent work with L. Dabrowski, where we investigate such constraints for the spectral triple of the Standard Model of elementary particles.

Koen van den Dungen: The fermionic action.

I will discuss the Lorentzian version of the fermionic action in non-commutative geometry, which is based on the use of Krein spaces instead of Hilbert spaces. This fermionic action correctly recovers the fermionic Lagrangians for standard examples of gauge theories, including the full Standard Model of particle physics. The description of these examples does not require a real structure (or charge conjugation), unless one includes Majorana masses for right-handed neutrinos, in which case the internal spaces also exhibit a Krein space structure. This talk is based on arXiv:1505.01939.

Michał Eckstein: Hidden causal structure of gauge theories.

Noncommutative geometry provides a robust framework for particle physics models. Gauge and scalar field theories emerge in a natural way from the geometry of the 'internal space' of the model. However, the standard approach is developed in the Euclidean signature, in which the Lorentzian aspects of the theory are invisible. Drawing from the concept of Lorentzian spectral triples I will show that noncommutative geometry puts constraints not only on the propagation speed in the space-time component, but also on the evolution in the internal space of the model. As an illustration, I will discuss the casus of the two-sheeted space-time and relate it to a relativistic quantum effect – the Zitterbewegung. The empirical consequences of the presented theory will also be discussed.

Shane Farnsworth: The underlying algebraic structure behind the standard model of particle physics.

Why is the standard model of particle physics (SM) the way it is? As will be described in an earlier talk by Latham Boyle, the NCG SM may be elegantly repackaged as an Eilenberg $*$-DGA. This construction recovers many of the traditional axioms of NCG and provides novel constrains on SM particle content. This construction also provides a novel geometric interpretation of electro-weak symmetry breaking and an interesting new contact point between geometry and quantization. In this talk I will flesh out many of the most interesting details of our construction which Latham will mention in his talk.

Farzad Fathizadeh: Modular forms in the spectral action of Bianchi-IX gravitational instantons.

In a succession of papers, physicists and mathematicians have achieved an explicit parametrization of Bianchi-IX gravitational instantons in terms of theta functions with characteristics. By exploiting the latter, in this talk I will shed light on a rationality phenomena in the spectral action of $SU(2)$-invariant Bianchi-IX metrics, by showing that each term in the expansion of the spectral action of the instantons gives rise to a modular form of weight 2 that can be explicitly written in terms of well-known modular forms, namely the Eisenstein series and the modular discriminant. An elegant proof of the rationality result will also be presented, which is based on expressing Seeley-de Witt coefficients as noncommutative residues of Laplacians. This talk is based on joint works with Wentao Fan and Matilde Marcolli.

Victor Gayral: From equivariant quantization to locally compact quantum groups.

In this talk, I will explain how a non-formal G-equivariant (or quasi-G-equivariant) quantization on a locally compact group G may, under good circumstances, allows one to construct a locally compact quantum group (in the sense of Kustermans-Vaes) deforming G, via a construction of a unitary dual two-cocycle on G or more generally via the construction of a manageable multiplicative unitary. Examples will be associated with Kählerian groups of negative sectional curvature (Pyatetskii-Shapiro's groups) and quotient groups of certain subgroups of the affine group of a non-Archimedean local field.

Magnus Goffeng: Detecting regularity using cyclic cocycles and singular traces.

We will discuss cyclic cocycles defined from weakly summable Fredholm modules and singular traces. These cocycles vanish under the periodicity operator S and admit explicit pre-images in Hochschild cohomology. The pairing with cyclic cohomology is in principle computable. A closed expression is in general not possible; already on closed manifolds the pairing involves computing singular traces of non-measurable operators. The main example involves Hölder continuous functions on a closed manifold where the cyclic cocycles 'detect' regularity and vanish on smooth functions.

José Gracia-Bondía: Light-like string localized quantum fields.

As gravitational waves provide a post-modern *Sidereus Nuncius* to scour the heavens, it is useful to reflect on the status of our perception of it. Non-commutative geometry has been termed spectral geometry, since its empirical root is the acknowledgment that we only sense the universe through very narrow light-fronts. Now, Einstein used to say that those who think they understand the light quantum are fooling themselves. Indeed there are few experiences more embarrassing to physicists writing textbooks that trying to sort out the clash between the relativity principle and the apparatus of quantum mechanics; a clash made much worse in the world of gluons and massive vector bosons. Surely, each science requires and engenders its own technologies, be they concrete or abstract tools: to deal with that clash we got the exotic herd of ghost fields, anti-fields, Nakanishi-Lautrup Lagrange multipliers, S-operators and whatnot, replacing the hapless Gupta-Bleuler 'formalism'. But tools can (and should) evolve even where there is little progress in basic understanding. In the talk I exhibit a new version of a recent technology for quantum field theory, preempting the clash without calling up the ghosts and opening new vistas in the elementary particle realm. In its framework the very concept of gauge transformation dissolves.

Peter Hochs: A fixed point theorem on non-compact manifolds.

For an elliptic operator on a compact manifold acted upon by a compact Lie group, the Atiyah-Segal-Singer fixed point formula expresses its equivariant index in terms of data on fixed point sets of group elements. This can for example be used to prove Weyl?s character formula. Using KK-theory, we extend the definition of the equivariant index to non-compact manifolds and prove a generalisation of the Atiyah-Segal-Singer formula, for group elements with compact fixed point sets. In one example, this leads to a relation with characters of discrete series representations of semisimple Lie groups. (This is joint work with Hang Wang.)

Jens Kaad: Morita equivalences of spectral triples.

Classically, two algebras are Morita equivalent when they are linked by a pair of finitely generated projective bimodules via the module tensor product. In particular two such algebras will have the same representation theory. At the C^*-algebraic level this concept was extended by Marc Rieffel using the interior tensor product of C^*-correspondences and the extra analytic flexibility then allows for the use of modules with infinitely many generators. The idea that Morita equivalent algebras should also have the same non-commutative geometry (meaning that they admit the same spectral triples) has been successfully applied to the algebraic case where the correspondence takes place via a finitely generated projective module. In this talk I will show how the unbounded Kasparov product can be applied to obtain Morita equivalence results for spectral triples via infinitely generated modules. As an application we shall see that the following pairs of objects admit the same (twisted) spectral triples: 1) two Riemannian manifolds with conformally equivalent metrics; 2) a hereditary subalgebra and its full algebra; 3) the crossed product of a discrete group acting on a manifold and the quotient manifold.

Giovanni Landi: Line bundles over noncommutative spaces.

We give a Pimsner algebra construction of noncommutative lens spaces as 'direct sums of line bundles' and exhibit them as 'total spaces' of certain principal bundles over noncommutative weighted projective spaces. For each quantum lens space one gets an analogue of the classical Gysin sequence which can be used to give explicit geometric representatives of the K-theory classes of the lens spaces.

Fedele Lizzi: Wick rotation, fermion doubling and Lorentz symmetry for the spectral action.

We deal with two features of the spectral action approach to the Standard Model: the fact that the model requires a Wick rotation to Lorentzian space and that it shows a quadrupling of the fermionic degrees of freedom. We show how the two issues are intimately related. We give a precise prescription to Wick rotate the Euclidean theory to the Lorentzian one and eliminate the extra degrees of freedom. This requires not only a projecting out of mirror fermions but also the elimination of remaining extra degrees of freedom. The remaining doubling has to be removed in order to recover the correct Fock space of the physical (Lorentzian) theory. To have a $\mathrm{S}pin(4)$-invariant Euclidean theory and a $\mathrm{S}pin(1,3)$-invariant Lorentzian theory, such an elimination must be performed after the Wick rotation. Some connections with Clifford symmetries will be mentioned at the end, time permitting.

Thierry Masson: Generalized connections and Higgs fields on Lie algebroids.

Connections in noncommutative geometry are well known to naturally produce, in many situations, Higgs fields. We show that this feature is shared with a correct notion of generalized connections on transitive Lie algebroids. Comparison with noncommutative geometry will be given. This leads to a better understanding of the key mathematical structure at the heart of the construction of gauge field models with Higgs fields and spontaneous symmetry breaking.

Bram Mesland: Hecke operators and K-homology of arithmetic groups.

Cohomology of arithmetic groups and its structure as a Hecke module plays a prominent role in modern number theory. Classically the cohomology of an arithmetic group Γ can be studied geometrically through its action on the associated global symmetric space X. In low dimensions, such actions produce non-compact hyperbolic manifolds as quotient spaces, as well as dynamically complicated actions on the boundary of X. In joint work with Haluk Sengun (Sheffield), we show that the cohomology of Γ, as a Hecke module, can be captured by the K-groups of a certain noncommutative C*-algebra which encode the action of Γ on X as well as its the boundary. The Hecke operators can be rigidly defined as explicit classes in KK-theory, acting on the relevant K-groups in a way compatible with Morita equivalence and boundary maps. This provides a uniform framework to study the K-homology of arithmetic groups.

Anna Pachoł: Hopf algebroids, Drinfeld twists and quantum space-times.

The idea of quantum space-time contributes to one of the approaches to Quantum Gravity. The space-time coordinates are no longer the classical variables but elements of a noncommutative 'coordinate algebra' as in quantum theory. In my talk I will introduce the mathematical framework for quantum space-times within the Hopf algebra formalism. In this picture, Hopf algebras are thought of as generalizations of symmetry groups of the underlying space-time and the space-time is a Hopf module algebra over the Hopf algebra of differential operators. Both algebras can undergo a deformation procedure. I will present the Drinfeld twist techniques, which are particularly useful in this quantization approach. Different constructions within the Hopf-algebraic framework leading to physically interesting cases will be shown as well. As one of them I will mention the crossed (smash) product construction combining twisted Hopf algebra and its module into a new algebra which, in fact, does not depend on the twist. By generalizing this picture to Hopf algebroids (i.e. Hopf algebras over noncommutative rings) one can turn the cross product algebra into a Hopf algebroid in a twist-dependent way. The Hopf algebroid framework introduces the new type of quantum spaces whose corresponding phase space has a bialgebraic structure.

Mairi Sakellariadou: Spectral Action and Cosmology.

I will first discuss some cosmological consequences of the bosonic spectral action based on a cut-off regularisation. Considering the spectral action of a noncommutative torsion geometry, I will then show that the Hamiltonian is bounded from below, a result that guarantees the linear stability of the theory. Finally, I will propose a novel definition of the bosonic spectral action using zeta function regularization, in order to address the issues of renormalizability and spectral dimensions.

Patrizia Vitale: The Gribov problem in noncommutative QED.

It is shown that in the noncommutative version of QED, Gribov copies induced by the noncommutativity of space-time appear in the Landau gauge. This is a genuine effect of noncommutative geometry which disappears when the noncommutative parameter vanishes. A similar phenomenon manifests itself in noncommutative scalar field theory, where local automorphisms of the star product give rise to Gribov-like copies.