spectral action and cosmology

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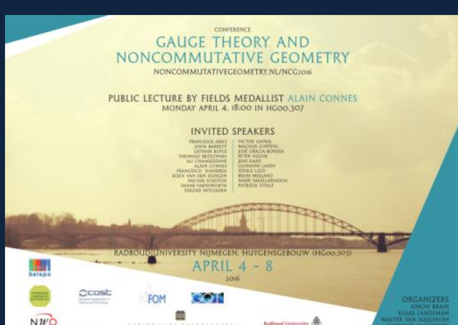


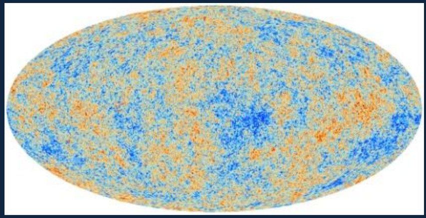
QSPACE

outline

- motivation
- noncommutative spectral geometry in a nutshell
- gravitational sector of the bosonic spectral action: cosmology
- linear stability of the bosonic spectral action
- shortcomings of the cutoff bosonic spectral action & a new regularisation proposal: the ζ spectral action
- open questions & concluding remarks

motivation

classical cosmology is built upon general relativity (GR) and the cosmological principle



but

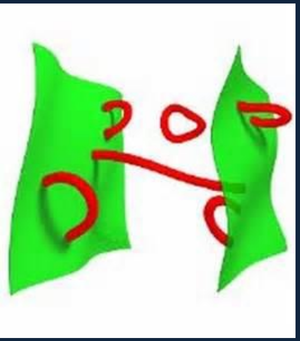
- GR is a classical effective theory valid at low energies

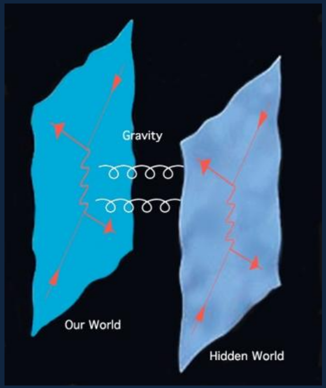
- the assumption of a continuous spacetime characterised by homogeneity and isotropy on large scales is valid at low energies

to describe the physics near the big bang, a quantum gravity (QG) theory with the appropriate space-time geometry is needed

quantum gravity (QG)

string/M-theory

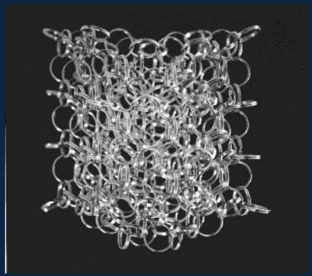


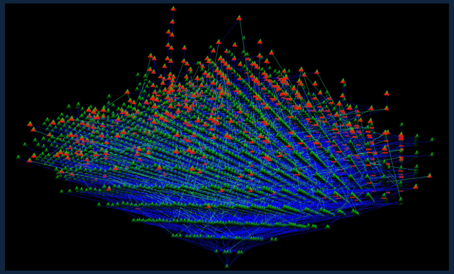


matter consists of
1-dim objects (strings)

matter is the important
ingredient

- loop QG
- euclidean QG
  (e.g., causal dynamical triangulations)
- group field theory





space is not infinitely divisible, but has
a granular structure: *quanta of space*

matter part is just added ...

top-down approach

noncommutative spectral geometry approach

QG : spacetime is wildly noncommutative manifold at very high energies

at an intermediate scale, the algebra of coordinates is only a mildly noncommutative algebra of matrix valued functions

if suitably chosen $\Rightarrow$ *standard model (SM) coupled to gravity*

bottom-up approach

guess small-scale structure of ST from knowledge at EW scale

to construct a quantum theory of gravity coupled to matter , gravity-matter interaction is the most important aspect for dynamics

noncommutative spectral geometry in a nutshell

SM: a phenomenological model which dictates spacetime geometry

$\mathcal{M} \times \mathcal{F}$

4-dim ST with an internal kaluza-klein space attached to each point; the 5th dim is a discrete, 0-dim space

spectral triple

$$M \times F := (C^\infty(M, \mathcal{A}_F), L^2(M,S) \otimes \mathcal{H}_F, \not{D} \otimes \mathbb{I} + \gamma_5 \otimes D_F)$$

$\mathcal{A} = C^\infty(\mathcal{M})$ $\mathcal{H}$ $-i\gamma^\mu \nabla^s_\mu$

$D_\mathcal{F}$ 96 x 96 matrix

8 fermions (electrons, neutrinos, up & down quarks with 3 colors each) for 2 chiralities for N=3 families + antiparticles

in terms of 3x3 Yukawa mixing matrices and a real constant responsible for neutrino mass terms

$$D_F = \begin{pmatrix} S & T^* \\ T & \bar{S} \end{pmatrix}$$

$$\mathcal{A}_\mathcal{F} = M_a(\mathbb{H}) \oplus M_k(\mathbb{C})$$

$$S = \begin{pmatrix} & \begin{bmatrix} \Upsilon_\nu & \\ & \Upsilon_e \end{bmatrix} & & \\ \begin{bmatrix} \Upsilon_\nu^* & \\ & \Upsilon_e^* \end{bmatrix} & & & \\ & & & \begin{bmatrix} \Upsilon_u \otimes 1_3 & \\ & \Upsilon_d \otimes 1_3 \end{bmatrix} \\ & & \begin{bmatrix} \Upsilon_u^* \otimes 1_3 & \\ & \Upsilon_d^* \otimes 1_3 \end{bmatrix} & \end{pmatrix} \quad T = \begin{pmatrix} \begin{bmatrix} 0 & & & \\ & 0 & & \\ & & \Upsilon_R & \\ & & & 0 \end{bmatrix} & \\ & 0_4 \end{pmatrix}$$

$$k = 2a$$
$$k = 4$$

chamseddine, connes (2007)

$$\mathcal{A}_{\mathcal{F}} = M_a(\mathbb{H}) \oplus M_k(\mathbb{C})$$

$$k = 2a$$

physical picture of the doubling of the algebra:

a geometric space (space-time) formed by 2 copies (branes) of 4dim manifold

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physical picture of the doubling of the algebra:

a geometric space (space-time) formed by 2 copies (branes) of 4dim manifold

- necessary to accommodate gauge symmetries, needed to describe the SM

- related to dissipation, hence to information loss, thus containing the seeds of quantisation (*following 't hoft's conjecture*)

- can explain neutrino mixing

sakellariadou, stabile, vitiello, PRD 84 (2011) 045026

gargiulo, sakellariadou, vitiello, EPJ C 74 (2014) 2695 spectral action principle:

$$A = \sum_j a_j[\mathcal{D}_{\mathcal{F}}, b_j] \quad , \quad a_j, b_j \in \mathcal{A}_{\mathcal{F}}$$

$$\mathcal{D}_A = \mathcal{D}_{\mathcal{F}} + A + \epsilon' J A J^{-1}$$

the action functional depends only on the spectrum of the (generalised) Dirac operator and is of the form:

$$Tr\left(f\left(D_A^2 / \Lambda^2\right)\right)$$

bosonic part

$f(x) = 1$ if $x \leq \Lambda$ $\quad$ $f(x) = e^{-x}$

evaluate trace with heat kernel techniques

$$\mathrm{Tr}\left(f\left(\frac{D_A^2}{\Lambda^2}\right)\right) \sim 2f_4\Lambda^4 a_0(D_A{}^2) + 2f_2\Lambda^2 a_2(D_A{}^2) + f(0)a_4(D_A{}^2) + O(\Lambda^{-2})$$

chamseddine, connes (1996,1997)

spectral action principle:

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the action functional depends only on the <u>spectrum</u> of the (generalised) Dirac operator and is of the form:

$$Tr\left(f\left(D_A^2/\Lambda^2\right)\right) + \frac{1}{2}\langle J\Psi, \mathcal{D}_A\Psi\rangle \ , \ \Psi \in \mathcal{H}_{\mathcal{F}}^+$$

evaluate trace with heat kernel techniques

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chamseddine, connes (1996,1997)

the rest follows after a long calculation ...

$$S_\Lambda = \int d^4x\sqrt{g}\,\Big(A_1\Lambda^4 \;+A_2\Lambda^2\left(\frac{5}{4}R - 2y_t^2H^2 - M^2\right)$$
$$+A_3\left(g_2^2W^\alpha_{\mu\nu}W^{\alpha\;\mu\nu} + g_3^2G^a_{\mu\nu}G^{a\;\mu\nu} + \frac{5}{3}g_1^2B_{\mu\nu}B^{\mu\nu}\right)$$
$$+\text{other }\mathcal{O}(\Lambda^0)\;\;+\mathcal{O}\left(\Lambda^{-2}\right)\Big)$$

$$\frac{5}{3}g_1^2(\Lambda) = g_2^2(\Lambda) = g_3^2(\Lambda) \;\longrightarrow\; \Lambda \sim \left(10^{14} - 10^{17}\right)\ \text{GeV}$$

use RGE to get predictions for SM

chamseddine, connes, marcolli (2007)

compatible with 126 GeV higgs mass

stephan (2009)

connes, chamseddine (2012)

chamseddine, connes, van suijlekom (2013)

devastato, lizzi, martinetti (2013)

gravitational terms coupled to matter:

$$\mathcal{S}^{\mathrm{E}}_{\text{bosonic}} = \int \Big(\frac{1}{2\kappa_0^2} R + \alpha_0 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \gamma_0 + \tau_0 R^\star R^\star + \frac{1}{4} G^i_{\mu\nu} G^{\mu\nu i} + \frac{1}{4} F^\alpha_{\mu\nu} F^{\mu\nu\alpha} + \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + \frac{1}{2} |D_\mu \mathbf{H}|^2 - \mu_0^2 |\mathbf{H}|^2 - \xi_0 R |\mathbf{H}|^2 + \lambda_0 |\mathbf{H}|^4 \Big) \sqrt{g}\, d^4x \,,$$

$$\begin{aligned}
\kappa_0^2 &= \frac{12\pi^2}{96 f_2 \Lambda^2 - f_0 \mathfrak{c}} \,, \\
\alpha_0 &= -\frac{3 f_0}{10\pi^2} \,, \\
\gamma_0 &= \frac{1}{\pi^2} \left(48 f_4 \Lambda^4 - f_2 \Lambda^2 \mathfrak{c} + \frac{f_0}{4} \mathfrak{d} \right) , \\
\tau_0 &= \frac{11 f_0}{60\pi^2} \,, \\
\mu_0^2 &= 2\Lambda^2 \frac{f_2}{f_0} - \frac{\mathfrak{e}}{\mathfrak{a}} \,, \\
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\lambda_0 &= \frac{\pi^2 \mathfrak{b}}{2 f_0 \mathfrak{a}^2} \,;
\end{aligned}$$

bare action *à la* wislon

$\mathfrak{a}, \mathfrak{b}, \mathfrak{c}, \mathfrak{d}, \mathfrak{e}$ describe possible choices of $\mathcal{D}_{\mathcal{F}}$

chamseddine, connes, marcolli (2007)

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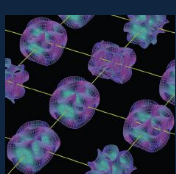
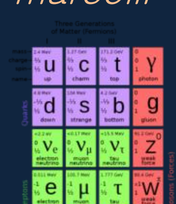
- EH action with a cosmological term
- topological term
- conformal gravity term with the weyl curvature tensor
- conformal coupling of higgs to gravity

the coefficients of the gravitational terms depend on yukawa parameters of the particle physics content

chamseddine, connes, marcolli (2007)



NCSG offers a geometric interpretation of the SM coupled to gravity



the gravitational sector of the bosonic spectral action:
cosmology

approach:

consider the gravitational sector of the theory as an extended gravity model, and address some cosmological questions

- this implies the hypothesis that one is able to perform wick rotation to imaginary time and get lorentzian signature

- expansion is valid when fields & their derivatives are small wrt Λ
  weak-field approximation

- higher derivative theory: is it plagued by linear instability (appearance of ghosts) ?

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cutoff spectral action

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\end{aligned}$$

valid at the cutoff scale Λ

equations of motion (background equations)

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R + \frac{1}{\beta^2}\delta_{\rm cc}\left[2C^{\mu\lambda\nu\kappa}_{\ ;\lambda;\kappa} + C^{\mu\lambda\nu\kappa}R_{\lambda\kappa}\right] = \kappa_0^2 \delta_{\rm cc} T^{\mu\nu}_{\rm matter}$$

$$\beta^2 \equiv -\frac{1}{4\kappa_0^2\alpha_0}$$

$$\delta_{\rm cc} \equiv [1 - 2\kappa_0^2\xi_0 \mathbf{H}^2]^{-1}$$

$$\alpha_0 = \frac{-3f_0}{10\pi^2}$$

nelson, sakellariadou, PRD 81 (2010) 085038

equations of motion (background equations)

- low-energy regime

i.e., neglect nonminimal coupling between geometry and higgs field

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at leading order, only in anisotropic and inhomogeneous spacetimes

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- high-energy regime

$$\boxed{R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = \kappa_0^2 \left[\frac{1}{1 - \kappa_0^2 |\mathbf{H}|^2/6} \right] T^{\mu\nu}_{\text{matter}}}$$

$$\mathcal{L}_{|\mathbf{H}|} = -\frac{R}{12}|\mathbf{H}|^2 + \frac{1}{2}|D^\alpha \mathbf{H}||D^\beta \mathbf{H}|\, g_{\alpha\beta} - \mu_0 |\mathbf{H}|^2 + \lambda_0 |\mathbf{H}|^4$$

nelson, sakellariadou, PRD 81 (2010) 085038

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i.e., neglect nonminimal coupling between geometry and higgs field

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- high-energy regime

coupling plays the role of an effective gravitational constant;
alternatively, the coupling increases the self-interaction of the higgs field

similarity with compactified string models and
with chameleon cosmology

nelson, sakellariadou, PRD 81 (2010) 085038

cutoff spectral action

$$\mathscr{S}^{\mathrm{E}} = \int \Bigg(\frac{1}{2\kappa_0^2} R + \alpha_0 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \gamma_0 + \tau_0 R^\star R^\star + \frac{1}{4} G^i_{\mu\nu} G^{\mu\nu i} + \frac{1}{4} F^\alpha_{\mu\nu} F^{\mu\nu\alpha}$$

bosonic

$$+ \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + \frac{1}{2} |D_\mu \mathbf{H}|^2 - \mu_0^2 |\mathbf{H}|^2 - \xi_0 R |\mathbf{H}|^2 + \lambda_0 |\mathbf{H}|^4 \Bigg) \sqrt{g}\, d^4x\,,$$

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linear perturbations around minkowski

$$\beta^2 = -\frac{1}{32\pi G \alpha_0}$$ *plays the role of a mass (>0)*

$$\frac{g_3^2 f_0}{2\pi^2} = \frac{1}{4} \qquad g_3^2 = g_2^2 = \frac{5}{3} g_1^2$$

propagation of GW

constraints from binary pulsars

nelson, ochoa, sakellariadou, PRD 82 (2010) 085021

PRL 105 (2010) 101602

cutoff spectral action

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$$\boxed{\left(\Box - \beta^2\right) \Box h^{\mu\nu} = \beta^2 \frac{16\pi G}{c^4} T^{\mu\nu}_{\text{matter}}}$$

with $\frac{\partial}{\partial x^\mu} T^\mu_\nu = 0$

nelson, ochoa,sakellariadou, PRD 82 (2010) 085021

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energy loss to gravitational radiation by orbiting binaries

$$\boxed{-\frac{\mathrm{d}\mathcal{E}}{\mathrm{d}t} \approx \frac{c^2}{20G} |\mathbf{r}|^2 \dot{h}_{ij} \dot{h}^{ij}}$$

nelson, ochoa, sakellariadou, PRL 105 (2010) 101602

cutoff spectral action

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$$\tau_0 = \frac{11 f_0}{60\pi^2} \,,$$

$$\mu_0^2 = 2\Lambda^2 \frac{f_2}{f_0} - \frac{\mathfrak{e}}{\mathfrak{a}} \,,$$

$$\xi_0 = \frac{1}{12} \,,$$

$$\lambda_0 = \frac{\pi^2 \mathfrak{b}}{2 f_0 \mathfrak{a}^2} \,;$$

linear perturbations around minkowski

$$\beta^2 = -\frac{1}{32\pi G \alpha_0}$$

plays the role of a mass (>0)

$$\frac{g_3^2 f_0}{2\pi^2} = \frac{1}{4} \qquad g_3^2 = g_2^2 = \frac{5}{3} g_1^2$$

$$\left(\Box - \beta^2\right) \Box h^{\mu\nu} = \beta^2 \frac{16\pi G}{c^4} T^{\mu\nu}_{\text{matter}}$$

with $\frac{\partial}{\partial x^\mu} T^\mu_\nu = 0$

energy loss to gravitational radiation by orbiting binaries

$$-\frac{d\mathcal{E}}{dt} \approx \frac{c^2}{20G} |\mathbf{r}|^2 \dot{h}_{ij} \dot{h}^{ij}$$

strong deviations from GR at a critical frequency scale:

$$2\omega_c \equiv \beta c \sim (f_0 G)^{-1/2} c$$

nelson, ochoa, sakellariadou, PRL 105 (2010) 101602

cutoff spectral action

$$\mathcal{S}^{E} = \int \left(\frac{1}{2\kappa_0^2} R + \alpha_0 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \gamma_0 + \tau_0 R^\star R^\star + \frac{1}{4} G^i_{\mu\nu} G^{\mu\nu i} + \frac{1}{4} F^\alpha_{\mu\nu} F^{\mu\nu\alpha} \right.$$

bosonic

$$\left. + \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + \frac{1}{2} |D_\mu \mathbf{H}|^2 - \mu_0^2 |\mathbf{H}|^2 - \xi_0 R |\mathbf{H}|^2 + \lambda_0 |\mathbf{H}|^4 \right) \sqrt{g}\, d^4x \,,$$

$$\kappa_0^2 = \frac{12\pi^2}{96 f_2 \Lambda^2 - f_0 \mathfrak{c}} \,,$$

$$\boxed{\alpha_0 = -\frac{3 f_0}{10\pi^2} \,,}$$

$$\gamma_0 = \frac{1}{\pi^2} \left(48 f_4 \Lambda^4 - f_2 \Lambda^2 \mathfrak{c} + \frac{f_0}{4} \mathfrak{d} \right) ,$$

$$\tau_0 = \frac{11 f_0}{60\pi^2} \,,$$

$$\mu_0^2 = 2\Lambda^2 \frac{f_2}{f_0} - \frac{\mathfrak{e}}{\mathfrak{a}} \,,$$

$$\xi_0 = \frac{1}{12} \,,$$

$$\lambda_0 = \frac{\pi^2 \mathfrak{b}}{2 f_0 \mathfrak{a}^2} \,;$$

restrict β by requiring the magnitude of deviations from GR to be less than the accuracy to which the rate of change of orbital period agrees with GR predictions

Binary	Distance (pc)	Orbital Period (hr)	Eccentricity	GR (%)
PSR J0737-3039	~ 500	2.454	0.088	0.2
PSR J1012-5307	~ 840	14.5	$< 10^{-6}$	10
PSR J1141-6545	> 3700	4.74	0.17	6
PSR B1916+16	~ 6400	7.752	0.617	0.1
PSR B1534+12	~ 1100	10.1	?	1
PSR B2127+11C	~ 9980	8.045	0.68	3

nelson, ochoa, sakellariadou, PRL 105 (2010) 101602

cutoff spectral action

$$\mathcal{S}^{E} = \int \left(\frac{1}{2\kappa_0^2} R + \alpha_0 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \gamma_0 + \tau_0 R^\star R^\star + \frac{1}{4} G^i_{\mu\nu} G^{\mu\nu i} + \frac{1}{4} F^\alpha_{\mu\nu} F^{\mu\nu\alpha} + \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + \frac{1}{2} |D_\mu \mathbf{H}|^2 - \mu_0^2 |\mathbf{H}|^2 - \xi_0 R |\mathbf{H}|^2 + \lambda_0 |\mathbf{H}|^4 \right) \sqrt{g}\, d^4x \,,$$

bosonic

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restrict β by requiring the magnitude of deviations from GR to be less than the accuracy to which the rate of change of orbital period agrees with GR predictions

Binary	Distance	Orbital	Eccentricity	GR
			0.617	0.1
			?	1
PSR B2127+11C	~ 9980	8.045	0.68	3

propagation of GW

$$\beta^2 = -\frac{1}{32\pi G \alpha_0}$$

constraints from binary pulsars

$$\beta > 7.55 \times 10^{-13} \mathrm{m}^{-1}$$

nelson, ochoa, sakellariadou, PRL 105 (2010) 101602

cutoff spectral action

$$\mathcal{S}^{\mathrm{E}} = \int \Big(\frac{1}{2\kappa_0^2} R + \alpha_0 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \gamma_0 + \tau_0 R^\star R^\star + \frac{1}{4} G^i_{\mu\nu} G^{\mu\nu i} + \frac{1}{4} F^\alpha_{\mu\nu} F^{\mu\nu\alpha}$$

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$$+ \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + \frac{1}{2} |D_\mu \mathbf{H}|^2 - \mu_0^2 |\mathbf{H}|^2 - \xi_0 R |\mathbf{H}|^2 + \lambda_0 |\mathbf{H}|^4 \Big) \sqrt{g}\, d^4x \,,$$

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$$\tau_0 = \frac{11 f_0}{60\pi^2} \,,$$

$$\mu_0^2 = 2\Lambda^2 \frac{f_2}{f_0} - \frac{\mathfrak{e}}{\mathfrak{a}} \,,$$

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linear perturbations around minkowski

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propagation of GW

constraints from GPB/LARES

Effect	Measured	Predicted
Geodesic precession	6602 ± 18	6606
Lense-Thirring precession	37.2 ± 7.2	39.2

lambiase, sakellariadou, stabile, JCAP 12 (2013) 020

cutoff spectral action

$$\mathcal{S}^{\mathrm{E}} = \int \Big(\frac{1}{2\kappa_0^2} R + \alpha_0 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \gamma_0 + \tau_0 R^\star R^\star + \frac{1}{4} G^i_{\mu\nu} G^{\mu\nu i} + \frac{1}{4} F^\alpha_{\mu\nu} F^{\mu\nu\alpha}$$

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$$+ \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + \frac{1}{2} |D_\mu \mathbf{H}|^2 - \mu_0^2 |\mathbf{H}|^2 - \xi_0 R |\mathbf{H}|^2 + \lambda_0 |\mathbf{H}|^4 \Big) \sqrt{g}\, d^4x \,,$$

e.o.m. for gyro-spin 3-vector

$$\frac{d\mathbf{S}}{dt} = \frac{d\mathbf{S}}{dt}\Big|_{\mathrm{G}} + \frac{d\mathbf{S}}{dt}\Big|_{\mathrm{LT}}$$

metric

$$ds^2 = -(1+2\Phi)dt^2 + 2\mathbf{A}\cdot d\mathbf{x}dt + (1+2\Psi)d\mathbf{x}^2$$

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$$ds^2 = -(1 + 2\Phi) dt^2 + 2\mathbf{A} \cdot d\mathbf{x} dt + (1 + 2\Psi) d\mathbf{x}^2$$

$$\frac{d\mathbf{S}}{dt}\Big|_{\mathrm{G}} = \mathbf{\Omega}_{\mathrm{G}} \wedge \mathbf{S} \quad \text{with} \quad \mathbf{\Omega}_{\mathrm{G}} = \frac{1}{2}[\nabla(\Phi - 2\Psi)] \wedge \mathbf{v}$$

$$\frac{d\mathbf{S}}{dt}\Big|_{\mathrm{LT}} = \mathbf{\Omega}_{\mathrm{LT}} \wedge \mathbf{S} \quad \text{with} \quad \mathbf{\Omega}_{\mathrm{LT}} = \frac{1}{2} \nabla \wedge \mathbf{A}$$

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lambiase, sakellariadou, stabile, JCAP 12 (2013) 020

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Geodesic precession	6602 ± 18	6606
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$$\Omega_{\mathrm{geodesic}} = \Omega_{\mathrm{geodesic(GR)}} + \Omega_{\mathrm{geodesic(NCG)}}$$

$$|\Omega_{\mathrm{geodesic(NCG)}}| \leq \delta\Omega_{\mathrm{geodesic}}$$

$$\delta\Omega_{\mathrm{geodesic}} = 18 \ \ \mathrm{mas/y}$$

cutoff spectral action

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$$\begin{aligned}
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linear perturbations around minkowski

$$\beta^2 = -\frac{1}{32\pi G \alpha_0}$$

plays the role of a mass (>0)

$$\frac{g_3^2 f_0}{2\pi^2} = \frac{1}{4} \qquad g_3^2 = g_2^2 = \frac{5}{3} g_1^2$$

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$$\beta > 7.1 \times 10^{-5} \mathrm{m}^{-1}$$

lambiase, sakellariadou, stabile, JCAP 12 (2013) 020

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$$+ \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + \frac{1}{2} |D_\mu \mathbf{H}|^2 - \mu_0^2 |\mathbf{H}|^2 - \xi_0 R |\mathbf{H}|^2 + \lambda_0 |\mathbf{H}|^4 \Big) \sqrt{g}\, d^4 x \,,$$

linear perturbations around minkowski

$$\beta^2 = -\frac{1}{32\pi G \alpha_0}$$

plays the role of a mass (>0)

constraints from torsion balance

modifications by NCSG action to the newtonian potentials are similar to those by a fifth-force through a potential

$$V(r) = -\frac{GMm}{r}\left(1 + \alpha e^{-r/\lambda}\right)$$

$$\lambda = \beta^{-1}$$

$$\alpha \sim \mathcal{O}(1)$$

eöt-wash and irvine experiments $\lambda \lesssim 10^{-4}\mathrm{m}$ $\longrightarrow$ $\beta \geq 10^4\ \mathrm{m}^{-1}$

cutoff spectral action

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propagation of GW

constraints from torsion balance

$$\boxed{\beta \geq 10^4 \ \mathrm{m}^{-1}}$$

lambiase, sakellariadou, stabile, JCAP 12 (2013) 020

cutoff spectral action

bosonic

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\lambda_0 &= \frac{\pi^2 \mathfrak{b}}{2 f_0 \mathfrak{a}^2}\,;
\end{aligned}$$

could conformal coupling make the higgs field play the role of the inflaton?

running of higgs self-coupling at two-loops: slow-roll conditions satisfied but CMB constraints not satisfied (incompatibility with top quark mass)

nelson, sakellariadou, PLB 680 (2009) 263

buck, fairbairn, sakellariadou, PRD 82 (2010) 043509

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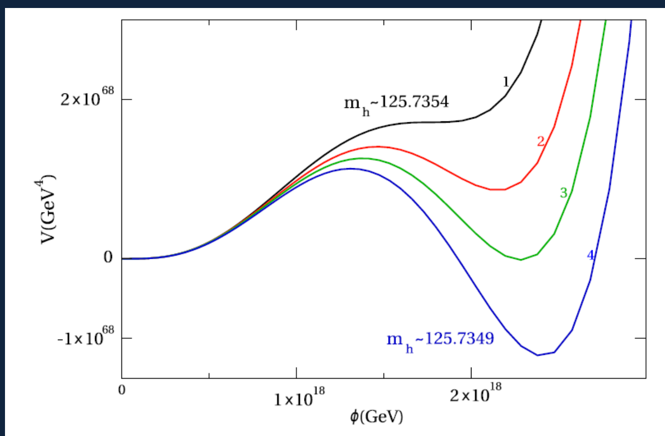

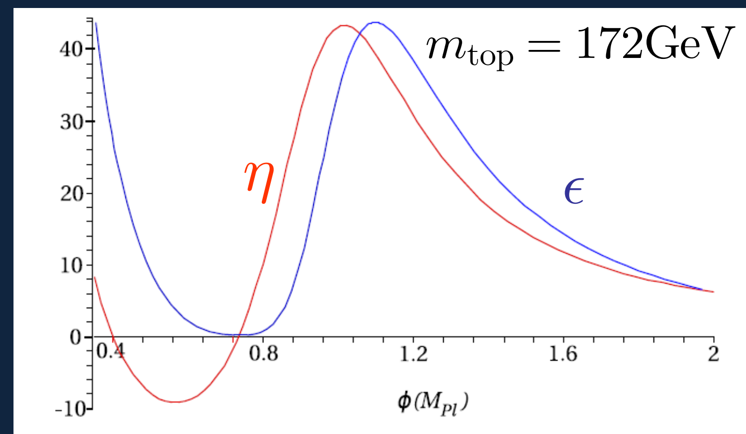

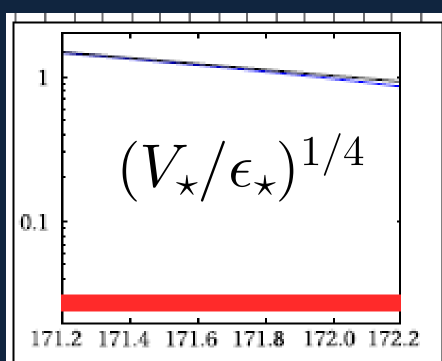

$$N \sim \epsilon^{-1/2} \mathrm{d}\phi$$

nelson, sakellariadou, PLB 680 (2009) 263

buck, fairbairn, sakellariadou, PRD 82 (2010) 043509

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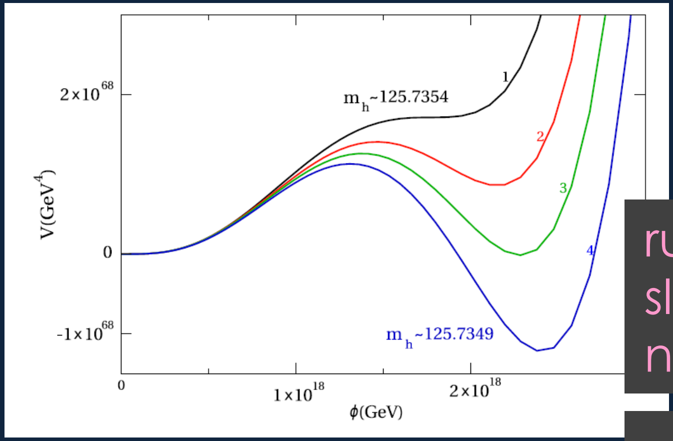
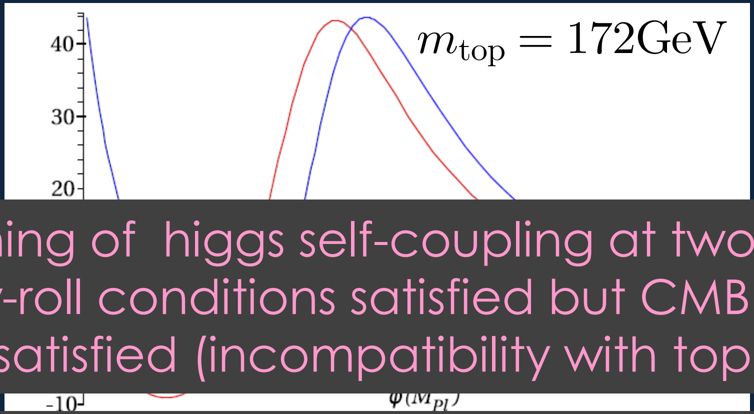
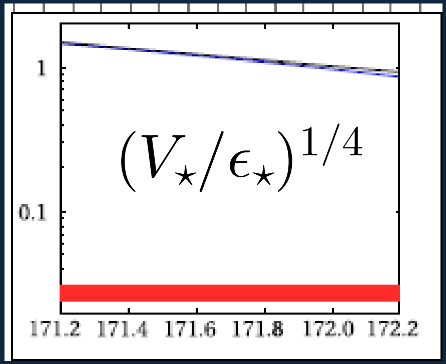




running of higgs self-coupling at two-loops:
slow-roll conditions satisfied but CMB constraints not satisfied (incompatibility with top quark mass)

valid for any scalar field conformaly coupled with the background (i.e., the σ- field)



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\xi_0 &= \frac{1}{12} \,, \\
\lambda_0 &= \frac{\pi^2 \mathfrak{b}}{2 f_0 \mathfrak{a}^2} \,;
\end{aligned}$$

could we find an inflationary mechanism?

the arbitrary mass scale in the spectral action for the Dirac operator can be made dynamical by introducing a dilaton field:

$$\mathcal{D}/\Lambda \rightarrow e^{-\Phi/2} \mathcal{D} e^{-\Phi/2}$$

$$\Phi = (1/f)\tilde{\sigma}$$

could $\bar{\sigma}$ be an inflaton?

$$\mathcal{S}_{\mathrm{GDH}} = \int \sqrt{G} \left[-\frac{1}{2\kappa_0^2} R + \frac{1}{2} \left(1 + \frac{6}{\kappa_0^2 f^2} \right) G^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + G^{\mu\nu} D_\mu H'^{*} D_\nu H' - V_0 \left(H'^{*} H' \right) \right] d^4x$$

chamseddine, connes (2006)

linear stability of the bosonic spectral action

higher derivative theory: is it plagued by linear instability (appearance of ghosts)?

sakellariadou, watcharangkool, PRD 93 (2016) 064034

higher derivative theory: is it plagued by linear instability (appearance of ghosts)?

- spectral action in the context of 4dim manifold with torsion (antisymmetric and yields the curvature tensor with the same symmetric properties as the riemannian curvature tensor), in vacuum

varying all of the connection fields, and not only the metric, weyl gravity transforms from a fourth order theory into a theory of conformal equivalent classes of solutions to GR, under the requirement that torsion vanishes

wheeler (2014)

sakellariadou, watcharangkool, PRD 93 (2016) 064034

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wheeler (2014)

⟹ the fourth order differential equations can be reduced to those of second order derived from vacuum GR, if and only if torsion vanishes

sakellariadou, watcharangkool, PRD 93 (2016) 064034

higher derivative theory: is it plagued by linear instability (appearance of ghosts)?

- spectral action in the context of 4dim manifold with torsion (antisymmetric and yields the curvature tensor with the same symmetric properties as the riemannian curvature tensor), in vacuum
- spectral action of an almost commutative torsion geometry

one cannot obtain the integrability condition in the presence of either fermion fields or scalar fields

there exists a class of almost commutative torsion geometry (a subset of spin connections) that leads to a hamiltonian that is bounded from below

→ the theory does not suffer from a linear instability

sakellariadou, watcharangkool, PRD 93 (2016) 064034

shortcomings of the cutoff bosonic spectral action
& a new regularisation proposal:
the ζ spectral action

expansion is valid when fields & their derivatives are small wrt Λ
weak-field approximation

the (cutoff) spectral action is calculated via the asymptotic heat kernel expansion

$$\mathrm{Tr}\left(f\left(\frac{D_A^2}{\Lambda^2}\right)\right) \sim 2f_4\Lambda^4 a_0({D_A}^2) + 2f_2\Lambda^2 a_2({D_A}^2) + f(0)a_4({D_A}^2) + O(\Lambda^{-1})$$

this can be divergent and generally speaking does not coincide with the spectral action

expansion is valid when fields & their derivatives are small wrt Λ

weak-field approximation

the ζ spectral action

$$S_\zeta \equiv \lim_{s\to 0} \mathrm{Tr}\, D^{-2s} \equiv \underbrace{\zeta(0, D^2)}_{\text{well-defined}} = a_4\left[D^2\right] = \int d^4x\, \sqrt{g}\, L$$

with $L(x) = a_4(D^2, x)$

for a laplace type operator D^2 *,*
the point s=0 is not a pole

kurkov, Izzi, sakellariadou, watcharangkool, PRD 91 (2015) 065013

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for a laplace type operator D^2*,*
the point s=0 is not a pole

the dimensionful constant M appearing in the position corresponding to the majorana mass in the dirac operator introduces the correct lower dimensonal operators

$$\begin{aligned} L(x) \;=\; & \alpha_1 M^4 + \alpha_2 M^2 R + \alpha_3 M^2 H^2 \\ & + \alpha_4 B_{\mu\nu} B^{\mu\nu} + \alpha_5 W^{\alpha}_{\mu\nu} W^{\mu\nu\,\alpha} + \alpha_6 G^{a}_{\mu\nu} G^{\mu\nu\,a} \\ & + \alpha_7 H \left(-\nabla^2 - \frac{R}{6}\right) H + \alpha_8 H^4 + \alpha_9 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \alpha_{10} R^* R^* \end{aligned}$$

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with $L(x) = a_4(D^2, x)$

for a laplace type operator D^2, *the point s=0 is not a pole*

the α_1, α_2, α_3 coefficients cannot be taken by the spectral action

i.e., the lower dimensional operators must be normalised by hand

$$\begin{aligned} L(x) \;=\; & \alpha_1 M^4 + \alpha_2 M^2 R + \alpha_3 M^2 H^2 \\ & + \alpha_4 B_{\mu\nu} B^{\mu\nu} + \alpha_5 W^{\alpha}_{\mu\nu} W^{\mu\nu\,\alpha} + \alpha_6 G^{a}_{\mu\nu} G^{\mu\nu\,a} \\ & + \alpha_7\, H\left(-\nabla^2 - \frac{R}{6}\right) H + \alpha_8 H^4 + \alpha_9 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \alpha_{10} R^* R^* \end{aligned}$$

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with $L(x) = a_4(D^2, x)$

well-defined

for a laplace type operator D^2,
the point s=0 is not a pole

only terms needed for the SM and einstein gravity

the lagrangian is an exact result

$$\begin{aligned} L(x) \;=\; & \alpha_1 M^4 + \alpha_2 M^2 R + \alpha_3 M^2 H^2 \\ & + \alpha_4 B_{\mu\nu} B^{\mu\nu} + \alpha_5 W^{\alpha}_{\mu\nu} W^{\mu\nu\,\alpha} + \alpha_6 G^{a}_{\mu\nu} G^{\mu\nu\,a} \\ & + \alpha_7\, H \left(-\nabla^2 - \frac{R}{6}\right) H + \alpha_8 H^4 + \alpha_9 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \alpha_{10} R^* R^* \end{aligned}$$

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to get (in the UV normalisation) the higgs quadratic term one needs a term in the dirac operator corresponding to the neutrino majorana mass; such nonzero term is also necessary to get the correct higgs mass

$$\bar\psi a_i \sigma(x)\psi \;\; ; \;\; i = 1, 2, 3$$

there are no dim 0 and dim 2 operators in the classical action

$$\begin{aligned} S_\zeta \;\; = \;\; & \int dx\,\sqrt{g}\left(\gamma_1 B_{\mu\nu}B^{\mu\nu} + \gamma_2 W^\alpha_{\mu\nu}W^{\mu\nu\,\alpha} + \gamma_3 G^a_{\mu\nu}G^{\mu\nu\,a} + \gamma_4\, H\left(-\nabla^2 - \frac{R}{6}\right)H\right. \\ & \left. +\gamma_5 H^4 + \gamma_6\,\sigma\left(-\nabla^2 - \frac{R}{6}\right)\sigma + \gamma_7\sigma^4 + \gamma_8 H^2\sigma^2\gamma_9 C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} + \gamma_{10}R^*R^*\right) \end{aligned}$$

dynamical generation of 3 scales upon quantisation

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the dirac operator has neutrino majorana mass terms of this kind in the context of the grand symmetry framework

devasto, lizzi, martinetti (2014)

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$$\psi^c(a_i\sigma(x) + M_i)\psi$$

these constant mass terms lead to the introduction of M^4, $M^2 H^2$ and $M^2 R$ terms in the action

$$\begin{aligned} S_\zeta \;\; = \;\; & \int dx\, \sqrt{g} \left(\gamma_1 B_{\mu\nu} B^{\mu\nu} + \gamma_2 W^{\alpha}_{\mu\nu} W^{\mu\nu\,\alpha} + \gamma_3 G^{a}_{\mu\nu} G^{\mu\nu\,a} + \gamma_4\, H \left(-\nabla^2 - \frac{R}{6} \right) H \right. \\ & \left. + \gamma_5 H^4 + \gamma_6\, \sigma \left(-\nabla^2 - \frac{R}{6} \right) \sigma + \gamma_7 \sigma^4 + \gamma_8 H^2 \sigma^2 \gamma_9 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \gamma_{10} R^* R^* \right) \end{aligned}$$

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M_i can be arbitrary small but nonzero in order to introduce countertems needed to normalise the cosmological constant, the quadraric higgs and einstein-hilbert terms

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- no higher dim (>4) operators — renormalisable and local
- no issues about asymptotic expansion and convergence
- S_ζ is purely spectral with no dependence on cutoff function

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spectral dimension

- cutoff spectral action

taking the full momentum-dependence of the propagators

→ spectral dimension vanishes for all spins

→ one needs a UV completion (e.g., asymptotic safety)

alkofer, saueressig, zanusso (2014)

high energy bosons do not propagate

kurkov, lizzi, vassilevich (2013)

spectral dimension

- ζ spectral action

actions for higgs scalar, gauge fields have same bahaviour in UV & IR

→ spectral dim = topological dim of manifold (=4)

gravitational spectral dim = 2

→ gravitational propagators decrease faster at infinity due to presence of fourth derivative(improvement on UV convergence)

there exists "low-energy" limit for which gravitational spectral dim=4

→ at very low energies dynamics does not feel weyl square terms

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open questions & concluding remarks

NCSG aims at explaining some conceptual issues of the SM, whilst it offers a geometrical framework to address physics at the QG regime, following the approach that the interaction between gravity and matter is the most important ingredient to define the dynamics

in particular, NCSG aims at defining the noncommutative algebra of observables of a QG theory

in NCSG, gravity and the SM fields are packaged into geometry and matter on a kaluza-klein noncommutative space and using experimental results at the electroweak scale, one tries to guess the small-scale space-time structure avoiding an *ad hoc* proposal

↓

NCSG offers a geometric interpretation of the SM coupled to gravity

NCSG lives by construction at the GUT scale, hence it provides a framework to build an early universe cosmological model

applying the spectral action within an almost commutative manifold, one gets gravity combined with yang-mills and higgs

- hypothesis that we are able to perform wick rotation to imaginary time so that we have lorentzian signature

- in the context of the cutoff spectral action
  expansion is valid when fields & their derivatives are small wrt Λ
  weak-field approximation

- in the context of the ζ spectral action
  find a dynamical generation of the three dimensionful fundamental constants, namely the cosmological constant, the higgs vacuum expectation value, and the gravitational constant

open question: inflation

neither the higgs field, nor the σ-field can lead to a successful inflation

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neither the higgs field, nor the σ-field can lead to a successful inflation

<u>comment</u>: *it is difficult to construct an inflationary scenario rooted in particle physics models*

none of the singlets of SM symmetries in the minimal set of SO(10) representations can satisfy the conditions necessary for a scalar field to be the inflaton, which has thus to be put as en extra field

cacciapaglia, sakellariadou, EPJ <u>74</u> (2014) 2779

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what about starobinsky-type inflation?

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what about starobinsky-type inflation?

$$S = \int \mathrm{d}^4x \sqrt{-g}\, \frac{M_{Pl}^2}{2} \left(R + \frac{1}{6M^2} R^2 \right)$$

conformally equivalent to a canonically normalised scalar field
with an exponentially flat "plateau" potential $V = V_0 \left(1 - e^{-\sqrt{2/3}\phi} \right)^2$

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$$S = \int \mathrm{d}^4x\sqrt{-g}\,\frac{M_{Pl}^2}{2}\left(R + \frac{1}{6M^2}R^2\right)$$

$M \sim 10^{-5} M_{Pl}$ small

conformally equivalent to a canonically normalised scalar field with an exponentially flat "plateau" potential

$$V = V_0\left(1 - e^{-\sqrt{2/3}\phi}\right)^2$$

COBE normalisation of the curvature perturbation requires $V_0 \sim M^2 M_{Pl}^2 \sim 10^{-10} M_{Pl}^4$

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$$\boxed{S \sim \int \sqrt{|g|} \left(\frac{f_4}{2\pi^2} \Lambda^4 + \frac{f_2}{24\pi^2} \Lambda^2 R - \frac{f(0)}{16\pi^2} ||C||^2 \right) d^4 x}$$

FLRW: weyl tensor vanishes $\Longrightarrow$ no modifications in einstein equations

starobinsky-type inflation cannot be applied

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- a scalar field in a beyond the SM approach ?

e.g., chamseddine, connes, van suijlekom (2013)

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pati-salam gauge group $\mathrm{G_{PS}} = \mathrm{SU(4)_C} \times \mathrm{SU(2)_R} \times \mathrm{SU(2)_L}$

$$4_{\rm C}\ 2_{\rm L}\ 2_{\rm R} \begin{cases} \xrightarrow{1} 3_{\rm C}\ 2_{\rm L}\ 2_{\rm R}\ 1_{\rm B-L} \begin{cases} \xrightarrow{1} 3_{\rm C}\ 2_{\rm L}\ 1_{\rm R}\ 1_{\rm B-L} \xrightarrow{2\ (2)} \mathrm{G_{SM}}\ (Z_2) \\ \xrightarrow{2'\ (2)} \mathrm{G_{SM}}\ (Z_2) \end{cases} \\ \xrightarrow{1} 4_{\rm C}\ 2_{\rm L}\ 1_{\rm R} \begin{cases} \xrightarrow{1} 3_{\rm C}\ 2_{\rm L}\ 1_{\rm R}\ 1_{\rm B-L} \xrightarrow{2\ (2)} \mathrm{G_{SM}}\ (Z_2) \\ \xrightarrow{2'\ (2)} \mathrm{G_{SM}}\ (Z_2) \end{cases} \\ \xrightarrow{1} 3_{\rm C}\ 2_{\rm L}\ 1_{\rm R}\ 1_{\rm B-L} \xrightarrow{2\ (2)} \mathrm{G_{SM}}\ (Z_2) \\ \xrightarrow{1\ (1,2)} \mathrm{G_{SM}}\ (Z_2) \end{cases}$$

jeannerot, rocher, sakellariadou, PRD 68 (2003) 103514

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$$
\mathrm{4_C\ 2_L\ 2_R} \left\{
\begin{array}{ll}
\xrightarrow{1} \mathrm{3_C\ 2_L\ 2_R\ 1_{B-L}} & \left\{ \begin{array}{l} \xrightarrow{1} \mathrm{3_C\ 2_L\ 1_R\ 1_{B-L}} \xrightarrow{2\ (2)} \mathrm{G_{SM}}\ (Z_2) \\ \xrightarrow{2'\ (2)} \mathrm{G_{SM}}\ (Z_2) \end{array} \right. \\
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\end{array}
\right.
$$

jeannerot, rocher, sakellariadou, PRD 68 (2003) 103514

$$\mathrm{SO(10)} \to \cdots \to \mathrm{G_{3,2,2,B-L}} \to \mathrm{G_{SM}} \times Z_2 \to \mathrm{SU(3)_C \times U(1)_Q} \times Z_2$$

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- a scalar field within the grand symmetry approach ?

  devasto, lizzi, martinetti (2014)

- the dilaton field ?

  chamseddine, connes (2006)

dank je